On the Use of a Lower Frequency in Finite-Difference Simulations for Urban Radio Coverage

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Abstract— **The Finite-Difference Time-Domain (FDTD) is not just one of the most accurate propagation prediction methods but also one of the most computationally intensive models, especially when simulating big areas such as urban environments. To overcome this problem, one common approach is to perform simulations at a much lower frequency than that of the real system, and to calibrate the simulation parameters to adjust the frequency response of the materials. In this study a theoretical analysis of the consequences of applying such frequency reduction is performed. To compare the theoretical study some numerical experiments are conducted and matched to the analytical results. Finally, the error of the simulations at lower frequencies is evaluated, allowing us to obtain an explanation for the observed behavior.**

I. INTRODUCTION

The Finite-Difference Time-Domain [1] [2] has been proven to be a very precise method for performing field predictions in small environments. Its accuracy has motivated several attempts to apply it to the prediction of radio coverage [3] [4], though one of the main limitations is still that FDTD implies the programming of a highly time-consuming algorithm.

To reduce the impact of anisotropic effects when performing FDTD calculations, it is a requirement [5] that the size of the spatial step be chosen several times smaller than the smallest wavelength to simulate, e.g. 10 times smaller for a velocity-anisotropy error lower than 0.9%. In addition, the time step is usually given as a function of the spatial step to avoid numerical dispersion and instability of the algorithm and its value is therefore also restricted by the wavelength to simulate [5]. For instance, a typical UMTS simulation at a frequency of approximately 2GHz would require a spatial step much smaller than $\lambda = 15cm$. When the objective is to make propagation predictions for network simulation, in urban areas of typically several square kilometers size, the computational requirements are usually far too high.

A common approach for this problem is to perform the simulation at a much lower frequency than that of the real system [6]. This way, the wavelength and the spatial step increase and the matrix that represents the propagation environment becomes smaller, reducing therefore the computational cost [7]. This process requires a calibration of the parameters of the different materials within the environment to account for its electrical properties at different frequencies. This way, a very accurate behavior of the materials can be achieved. But this approach also implies other effects that are mainly due to the relationship between the geometry of the simulated scenario and the wavelength of the simulation. The purpose of this article is to demonstrate the non-material-dependant distortion that occurs in the attenuation prediction obtained through FDTD simulations, when using a frequency lower than that of the system subject to study.

In section II an initial theoretical analysis of the potential results due to diffraction at lower frequencies is performed. Since this study is based on the empirical formulas provided by [8], it is necessary to perform some simulations in order to confirm or refuse the theoretical results. Section III compares the numerical and theoretical outcome in detail. Finally, in section IV we derive an expression to quantify the error in the attenuation calculation due to the use of a false frequency in Finite-Difference schemes.

II. THEORETICAL ATTENUATION RESULTS FOR KNIFE-EDGE DIFFRACTION

In urban environments the receiver will be most typically placed in a street (urban outdoor) where it does not have a direct Line-Of-Sight (LoS) to the transmitter. In this case the propagation phenomena that most contribute to carry the signal between both terminals are:

1) Reflections on the facades of buildings.
2) Diffractions on the edges of buildings.
3) Diffractions over rooftops (multiple screen diffraction [9]).
4) Scattering on the corners of buildings and urban furniture.

When FDTD simulations are performed at a lower frequency, the obtained reflections are not consistent with the results at the real frequency. This problem is usually solved by calibration of the reflecting properties of the different materials. On the other hand, diffractive effects are not so easily overcome due to the geometrical nature of diffraction itself. This implies that the simulations performed at lower frequencies will be subject to an error due to diffractive effects. In order to estimate such an error, a prediction of the attenuation at lower frequencies is necessary. Recommendation ITU-R P.526-10 [8] provides already a very precise empirical model

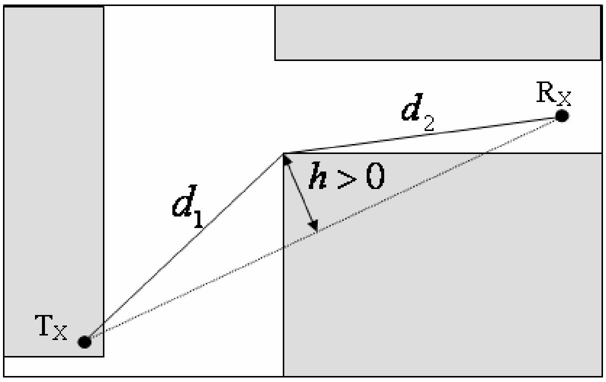

Fig. 1. Typical urban non LoS scenario.

for the evaluation of attenuation due to diffraction and it is therefore taken as a reference for the calculation of diffraction at different frequencies.

In this section edges are modeled as knife-edge obstacles. This way it is possible to compute the theoretical attenuation at different frequencies and compare it to the outcome of several numerical simulations.

The simplest case will be the study of the attenuation due to diffraction on a single edge of a building (see figure 1). In [8] h is defined as the obstruction of the obstacle above the line that joins transmitter and receiver. Since the formulas for the calculation of the attenuation are based on the Fresnel integrals, it is possible that positive attenuation exists even if $h < 0$ due to the obstruction of the Fresnel zones. The attenuation due to diffraction is mainly caused by the geometrical characteristics of the environment. This is why the electrical size of the wave plays a crucial role in the final value of the attenuation.

To study the effect of using a lower frequency on knife-edge diffraction over the edge of a building, we define the Frequency Reduction Factor (FRF) as:

$$FRF = \frac{f_{sim}}{f_{real}} \tag{1}$$

Where f_{sim} is the frequency at which the propagation prediction is calculated, and f_{real} is the real frequency of the system to simulate. We also define k as the proportion of the obstruction to the first Fresnel zone radius at the location of the obstacle:

$$k = \frac{h}{R_1} \tag{2}$$

As mentioned earlier, the main purpose of using a lower frequency for FDTD simulations is to reduce the spatial step and the size of the matrix representing the computational domain. That is why usually $f_{sim} < f_{real}$ and we will therefore restrict this study to the values within the range $0 < FRF < 1$.

According to [8], an accurate approximation for calculating the attenuation J due to diffraction at a given frequency is to apply the next set of simple formulas:

$$\nu = h\sqrt{\frac{2}{\lambda}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)} \tag{3}$$

$$J(\nu) = 6.9 + 20log\left(\sqrt{(\nu - 0.1)^2 + 1} + \nu - 0.1\right) \tag{4}$$

Where λ is the wavelength at the frequency of interest, and d_1 and d_2 are the distances to the edge as defined in figure 1. By applying (3) and (4) for different values of FRF and calculating the attenuation for an obstacle with different k when $FRF = 1$, we obtain the values displayed in figure 2. Since we would like to apply a frequency reduction to use the FDTD method in wireless networks simulations, we have chosen for this example a reference frequency of $f_{real} = 3.5GHz$, which is the working frequency of WiMAX (Worldwide Interoperability for Microwave Access) in Europe.

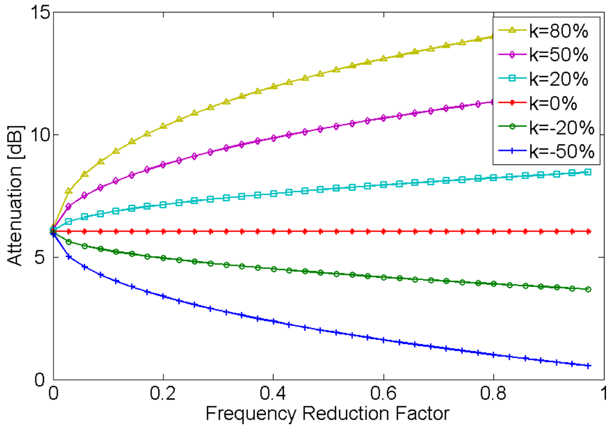

Fig. 2. Diffraction Attenuation at different frequencies and size of the obstacle.

By observing the results of figure 2 it is possible to distinguish different behaviors of the attenuation according to three different cases:

When $k < 0$ there exists a line of sight between transmitter and receiver but, depending on the frequency, the obstacle can still obstruct the first Fresnel zone. This is a case where the CPE (Customer Premises Equipment) lies in a street perpendicular to that of the Base Station but still has a line of sight (see figure 3). When that happens, there is an increase in the attenuation due to diffraction on the edge of a building at lower frequencies. To understand this effect we need to consider the radius R_n of the n^{th} Fresnel zone [8]:

$$R_n = 550\sqrt{\frac{nd_{T_x}d_{R_x}}{(d_{T_x} + d_{R_x})\, f}} \tag{5}$$

where f represents the frequency in MHz, and d_{T_x} and d_{R_x} are the distances (km) between transmitter and receiver at the point where the Fresnel radius (m) is calculated.

It is easy to see from (5) that the use of a lower frequency increases the size of the Fresnel zone. The required volume for the radiopropagation is thus bigger and the percentage of blocked volume increases with it. Figure 3 shows the change produced in the size of the Fresnel zone when decreasing the frequency (dashed line) and how the different obstacles gain importance at lower frequencies.

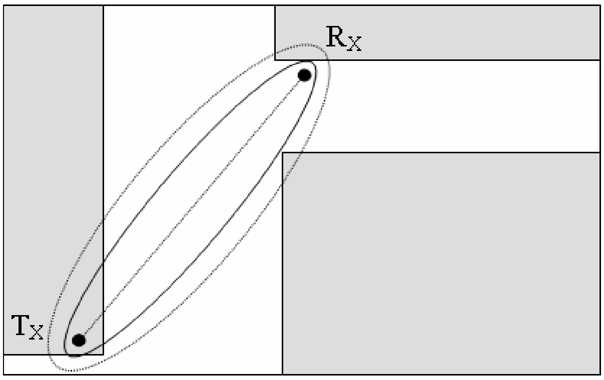

Fig. 3. Line-Of-Sight propagation with diffractive effects.

When $k = 0$ one half of the first Fresnel zone is always obstructed, which means that the attenuation will be always constant regardless of the radius of the Fresnel zone and therefore of the frequency.

As displayed in figure 4, $k > 0$ is a non Line-Of-Sight case at which there is always an obstructed half of the Fresnel zone. The other half of the ellipsoid will be more or less obstructed depending on its radius, i.e. the bigger the radius, the lower the obstruction. The attenuation due to knife-edge diffraction will hence decrease at lower frequencies, just as shown in figure 2. This is the most typical case in a wireless network within an urban environment because the terminals of the users rarely have a direct view of the Base Station.

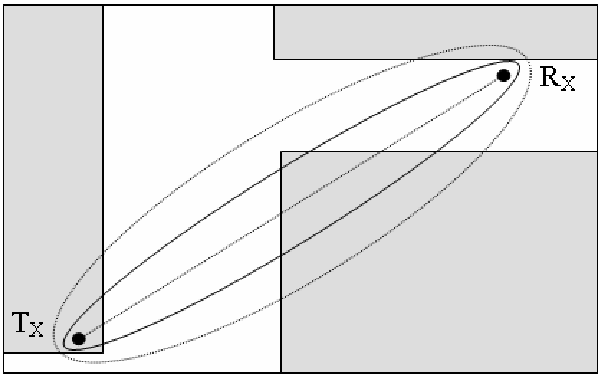

Fig. 4. Non Line-Of-Sight propagation due to diffraction.

The presented theoretical study indicates that a reduction of the simulation frequency can potentially lead to incorrect values of the field prediction due to diffraction on the edges of buildings. Moreover, the error in the results induced by applying a lower frequency would be different depending on the relative positions of both transmitter and receiver, what could lead to anisotropic distortion:

- In Line-Of-Sight cases with obstacles such as edges of buildings, the predicted attenuation would be overestimated.
- In non Line-Of-Sight cases (most common in urban environments) the attenuation due to diffraction would be underestimated.

III. NUMERICAL ATTENUATION RESULTS FOR KNIFE-EDGE DIFFRACTION

To verify the previous theoretical results, we have designed the test scenario presented in figure 5, in which the dark gray square represents a building whose edge will introduce diffraction effects. The chosen size of this scenario for our case of study was $S = 5m$ and the radius of the Fresnel zone midway from transmitter to receiver at $f_{real} = 3.5GHz$ is

$$R_1 = \frac{\sqrt{\lambda D}}{2}\bigg|_{\substack{f=3.5GHz \\ S=5m}} \approx 24.6cm \qquad (6)$$

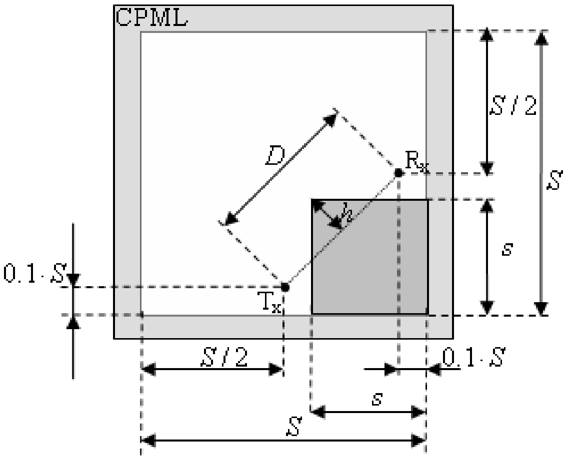

Fig. 5. Test Scenario for FDTD Simulations.

We have performed several numerical FDTD simulations for values s of the side of the building designed to produce the same degree k of obstruction as in the previous section (see figure 2). The FDTD method used was the traditional Yee algorithm in 2-D and TM_z mode as described in [5]. The transmitter is modeled as a sinusoidal single point soft source and the number of cells per wavelength was $N_\lambda = 10$ to reduce anisotropic effects and distortion of the wavefront. The chosen Courant number was $S = \frac{1}{\sqrt{2}}$ to avoid numerical dispersion and to guarantee the convergence and stability of the algorithm. These parameters lead to a 584x584 matrix at $f = 3.5GHz$ and much smaller matrices at lower frequencies. To simulate a reliable unbounded environment, we have placed a 40 cells thick absorbing boundary around the computational domain. This absorbing border follows the Convolutional Perfectly Matched Layer (CPML) [10] method, which we found very appropriate for our case since its formulation is completely independent of the algorithm applied to the computational domain.

The process followed to compute the attenuation due to knife-edge diffraction on the edge of a building is as follows:

1) An empty environment is simulated and the attenuation A_{empty} at the position of the receiver is recorded in dBs. This attenuation is only due to free space propagation.
2) The simulation is repeated with the building placed in its position according to figure 5 and the overall attenuation $A_{obstacle}$ is recorded
3) The reception point are chosen at positions where the main propagation phenomena are free-space and knife-edge diffraction. The attenuation in this case can be thus estimated by applying:

$$J_{diff} = A_{obstacle} - A_{empty} \tag{7}$$

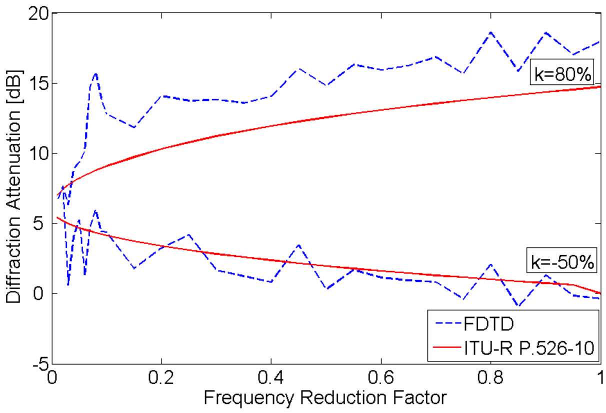

Fig. 6. Numerical and Theoretical Diffraction Predictions.

In figure 6 we display the calculated attenuations due to diffraction against the attenuation predicted by (4). The simulated obstructions in this case were $k = 80\%$ and $k = -50\%$ at the real frequency and we observe that the numerical results show approximately the same tendency as the theoretical results. The oscillation in the FDTD simulations can be explained due to several frequency-related effects, such as:

- Variation in the behavior of the materials.
- Different spatial and time steps.

We also notice that there exists a constant difference between the obtained FDTD attenuation and the theoretical one for non-LoS cases ($k = 80\%$). This is because (7) does not account for the losses introduced by the obstacle by means of energy absorption. Since the size of the obstacle remains constant along the FRF axis, such attenuation has always the same value, and we have confirmed that it only depends on the electrical properties of the obstacle. On the other hand, there is no such difference in the LoS cases, mainly because the LoS component is already strong enough and the obstacle does not impede as much energy as before from arriving at the receiver.

Despite of these effects, the knife-edge diffraction still dominates for the given scenario and the general tendency of the attenuation clearly agrees with the theoretical results.

IV. QUANTIFICATION OF THE ERROR

A numerical evaluation of the error due to the use of $FRF < 1$ can be deduced from the formulas given in [8]: Let's suppose for instance that we would like to obtain simulation results at a frequency of $f_{real} = 3.5GHz$. The attenuation due to diffraction on an isolated edge of a building would be $J_{real} \approx 14.72dBs$, when the obstacle is obstructing the 80% of the first Fresnel zone (non-LoS case) and it is located midway from transmitter and receiver. If we were to apply a Frequency Reduction Factor of $FRF = 0.1$ the attenuation would turn down to $J_{sim} \approx 9.11dBs$ which produces an absolute error of $E = J_{real} - J_{sim} = 5.62dBs$, meaning this that the real attenuation would be underestimated.

The attenuation due to diffraction is independent of the medium and relies only in the geometry of the scenario, e.g. other obstructions such as rounded-edge obstacles would produce different values of attenuation depending on the simulation frequency. This is mainly caused by the intrinsic nature of diffraction, which is the distortion of the wavefront when it encounters an obstacle whose size is on the order of the wavelength.

The obtained error for this example case might not seem remarkable compared to other sources of attenuation such as free space propagation, reflection losses and/or building penetration. However, the influence of this error will gain importance in much complex environments with many more knife-edges, such as city centers. Likewise, there exist many environments in which diffraction is the main propagation phenomenon, such as for example rooftops propagation [9] and at which the error magnitude would be remarkably higher.

By inserting (1) into (3) and (4) it is easy to derive a formula for the evaluation of the error produced when performing finite difference simulations with $FRF < 1$:

$$\nu_{real} = h\sqrt{\frac{2}{\lambda_{real}}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)} \tag{8}$$

$$\nu_{sim} = \sqrt{FRF}\nu_{real} \tag{9}$$

$$E = 20log\left(\frac{\sqrt{(\nu_{real} - 0.1)^2 + 1} + \nu_{real} - 0.1}{\sqrt{(\nu_{sim} - 0.1)^2 + 1} + \nu_{sim} - 0.1}\right) \tag{10}$$

A positive error indicates that the attenuation due to diffraction has been underestimated in the simulation, while a negative error indicates an overestimation.

In figure 7 we show the dependency of the error (10) with the Frequency Reduction Factor and the obstruction k. An interesting observation is that in the worst case, i.e. when the FRF is the lowest possible ($FRF = 0$ at the limit), the error in the prediction is limited and it depends only on the geometrical parameter ν at the real frequency. This maximum value of the error can also be predicted from figure 2, where we observe that the attenuations due to diffraction tend to a common value as FRF decreases. Such attenuation value can be calculated by making $\nu = 0$ in (4) and has a value of $J\,|_{FRF=0} \approx 6.03dBs$. This value arises from the formulation given in [8] and it is then straight to confirm from figure 6 that the magnitude of the error will be always limited by $|E| \leq |J_{real} - 6.03|$ with J_{real} in dBs.

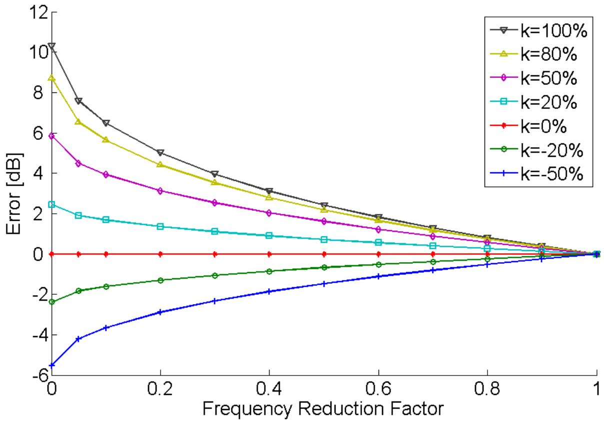

Fig. 7. Diffraction error when using a lower frequency in Finite-Difference Simulations.

V. CONCLUSIONS

The present study has verified that the use of a lower frequency in Finite-Difference calculations is subject to errors in the prediction of the attenuation due to diffraction at knife-edges. We have also confirmed that knife-edge diffraction is an accurate way of modeling the effects suffered by the wave when it encounters an edge of a building, and we have evaluated the value of such attenuation. Besides, following Recommendation ITU-R P.526-10 on "Propagation by Diffraction" [8], it is also possible to predict the attenuation values due to this phenomenon.

We have also presented the results of numerical simulations performed by means of the traditional Yee algorithm with a CPML absorbing border and we have measured the error produced when using a lower simulation frequency to perform Finite-Difference simulations.

Of uppermost interest is the fact that the error due to one single edge of a building is lower-limited by a value that does not depend on the simulation frequency if $k < 0$, and it is upper-limited by the opposite value if $k > 0$.

As a consequence of these results, we conclude that the reduction of the simulation frequency is subject to a diffraction error due to the increase of the electrical size of the wave and of the radius of the Fresnel zone. This error can not be easily overcome through the calibration of materials because it is basically a function of the geometry of the simulated scenario. Furthermore, since the attenuation due to diffraction will increase or decrease with FRF, depending on the existence of a line of sight to the transmitter, we believe that the reduction of the simulation frequency can even produce anisotropy on the calculation of the path loss.

As a consequence of this study, our current lines of research on Finite-Difference approaches, focus on the improvement of the performance of the FDTD algorithm by other means than reducing the simulation frequency. We are currently developing a FDTD platform for the prediction of radio coverage in big areas, based on alternative methods such as the Pseudospectral Time-Domain (PSTD) [11] [12] [13] and distributed and parallel [14] [4] computing approaches.

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